Accelerating SENSE-type MR image reconstruction algorithms with incremental gradients

Matthew J. Muckley[1,2], Douglas C. Noll[1,2], and Jeffrey A. Fessler[1,2]
[1]*Biomedical Engineering, University of Michigan, Ann Arbor, MI, United States,* [2]*Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, United States*

Target Audience: MR Physicists and engineers working on advanced image reconstruction methods.

Purpose: Advanced reconstruction methods have become an important research topic in recent years. Most of these methods seek to minimize a cost function, $\Psi(x)$. These cost functions are typically of the form $\Psi(x) = \|y - Ax\|_2^2 + R(x)$, where $\|y - Ax\|_2^2$ represents a data consistency term and $R(x)$ is a regularizer that preserves edges or promotes sparsity. Algorithms minimizing $\Psi(x)$ involve computing a term of the form $A^H(y - Ax)$, which is the gradient of the data consistency term, $\Phi(x) = \|y - Ax\|_2^2$. The computational cost of computing this term increases dramatically in non-Cartesian settings with SENSE-type parallel reconstructions. We propose an incremental gradient method for computing this term efficiently that can be implemented with almost any other algorithm with minimal modification of existing code. We demonstrate that the method can give four-fold acceleration when minimizing a "low rank plus sparse" (L+S) cost function,[1] which has been proposed for undersampled DCE MRI reconstruction.

Methods: Incremental gradient methods are based on the assumption that the overall function gradient can be expressed as the sum of M more computationally simple gradients, i.e. $\nabla\Phi(x) = \sum_{m=1}^{M} \nabla\phi_m(x)$, which applies to parallel MRI. We then make the simple approximation that $\nabla\Phi(x) \approx M\nabla\phi_m(x)$ at any given iteration of the algorithm. Since $\nabla\phi_m(x)$ requires less compute time than $\nabla\Phi(x)$, the algorithm is accelerated. These methods converge to a limit cycle around the true solution of the cost function under certain conditions,[2] but in early stages of the algorithm dramatic accelerations are realized. To demonstrate the acceleration, we considered the following L+S cost function for DCE MRI: $\Psi(L, S) = \|Y - A(L + S)\|_F^2 + \|L\|_* + \|S\|_1$. Majorizing the data fit term and minimizing leads to the iterative soft thresholding algorithm (ISTA) previously proposed for solving the L+S problem.[1] This algorithm requires the computation of a term of the form $A^H(Y - A(L + S))$ at each iteration. We replaced this term with $\alpha_j M A_m^H(Y_m - A_m(L + S))$, where m denotes a group of receive coils and α_j is a sequence that decreases to zero as a function of iteration count j. For the results here, we used $M = 4$. The group of receive coils should be chosen such that the gradient over the subset best approximates the full gradient. This can be done by choosing coils that evenly surround the image for each subset. The α_j parameter is present to help ensure decreasing step sizes, which have been shown to ensure convergence under certain conditions.[2] We measured convergence speed by comparing the iterates to a "converged solution" by running the original ISTA for 3,000 iterations. The DCE MRI data set (courtesy Ricardo Otazo at http://cai2r.net/research/ls-reconstruction) used golden angle radial sampling with 12 receive coils and an undersampling factor of 48. The algorithms produced a set of 128-by-128 images with 28 time points.

Results: Figure 1 shows that the incremental gradient method dramatically accelerates convergence in clock time by a factor of 4. The images shown in Figure 2 were obtained after 50 seconds of computation time using the incremental gradient method and the ISTA method. The ISTA images display streaking and noise artifacts that are significantly reduced in the incremental gradient methods.

Discussion: SENSE-type parallel reconstructions are a natural application for the methods described here. We expect the benefits of using incremental gradient methods to be greatest in applications with many receive coils. Incremental gradient techniques can be implemented with little modification of existing code, but lead to dramatic accelerations in convergence speed. The techniques described here provide computational advantages similar to those of coil compression, but incremental gradients do not require the discarding of any data. Although we illustrated the usefulness of this method with DCE MRI data, we suspect that similar benefits could be gained with any MRI application that uses multiple receive coils.

References: 1. R. Otazo et al., Submitted MRM 2013, 2. Z. Q. Luo., Neural Comput. 1991.

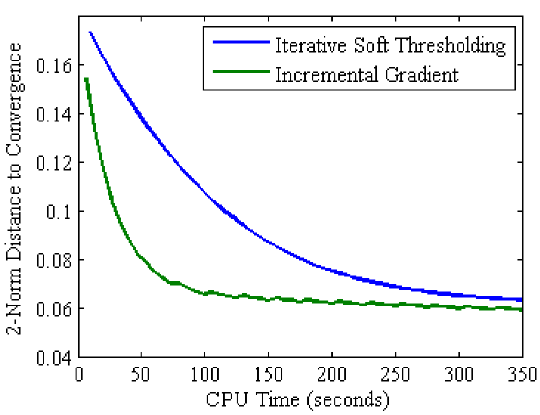

Figure 1: Convergence plot showing faster convergence of incremental gradients approach compared to IST[1].

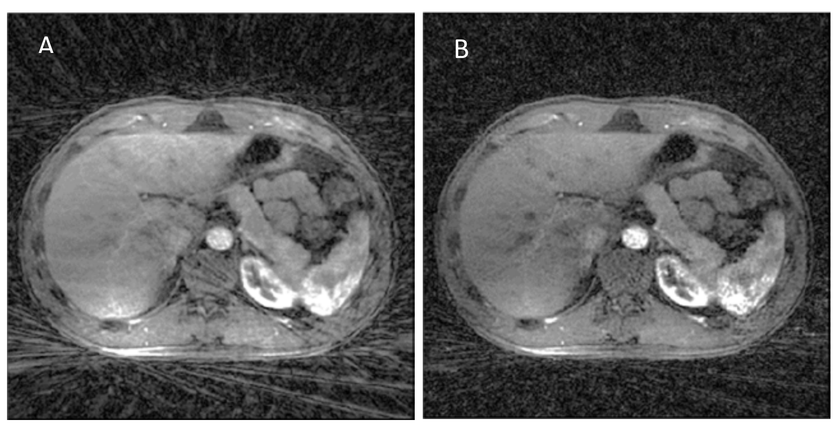

Figure 2: Images after 50 seconds of computation time, showing streaking artifacts in ISTA reconstruction (A) that are not present in the incremental gradient reconstruction (B).